Mode-mixing quantum gates and entanglement without particle creation in periodically accelerated cavities

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(Dated: October 2012)

We show that mode-mixing quantum gates can be produced by non-uniform relativistic acceleration. Periodic motion in cavities exhibits a series of resonant conditions producing entangling quantum gates between different frequency modes. The resonant condition associated with particle creation is the main feature of the dynamical Casimir effect which has been recently demonstrated superconducting circuits. We show that a second resonance, which has attracted less attention since it implies negligible particle production, produces a beam splitting quantum gate leading to a resonant enhancement of entanglement which can be used as the first evidence of the dynamical Casimir effect in mechanical oscillators. We propose a desktop experiment where the frequencies associated with this second resonance can be produced mechanically.

PACS numbers: 04.62.+v, 42.50.Xa, 42.50.Dv, 42.50.Pq

Introduction.— In relativistic quantum field theory, the particle content of a quantum state is affected by the evolution of the spacetime, including the motion of any boundaries. Further, the very notion of a "particle" depends on the motion of an observer. In flat spacetime, celebrated examples are the thermality seen in Minkowski vacuum by uniformly accelerated observers, known as the Unruh effect [1, 2], and the creation of particles by moving boundaries, known as the dynamical (or non-stationary) Casimir effect (DCE) [3, 4]. In curved spacetime, a celebrated example is the Hawking radiation emitted by black holes [5]. The DCE is related to a fundamental prediction by Fulling and Davies that a non-uniformly accelerated mirror will excite photons out of the vacuum [6]. It was later realised that this effect may be significantly enhanced if, instead of a simple mirror, a cavity is used in which one or both of the mirrors are in motion [7]. The simplest situation in which to observe the DCE is that of a cavity oscillating sinusoidally with frequency ω_r. The DCE is predicted to exhibit a fundamental resonance condition for the production of quantum entangled photon-pairs, $\omega_r = \omega_1 + \omega_2$, where $\omega_{1,2}$ are the two entangled photon frequencies [7]. The actual number of photons predicted for a mechanically oscillating cavity is strongly limited ($\sim 10^{-9}$ photons/second) by the maximum achievable ω_r. For this reason a number of alternative systems that also exhibit a periodically varying boundary of some kind have been proposed with the aim of enhancing the DCE. Examples are superconducting SQUID mirrors, Bose-Einstein condensates (producing phonon pairs) and cavities controlled using nonlinear optics [3, 4, 8, 9]. Notwithstanding recent breakthroughs, the DCE remains an extremely difficult effect to observe and study experimentally.

In this letter we consider the general case of a rigid cavity undergoing an arbitrary (mechanically induced) acceleration. In the specific cases of a linear sinusoidal or a uniform circular motion, we show that the mode mixing resonance condition, $\omega_r = |\omega_1 - \omega_2|$ [10], can be brought significantly below the DCE photon generation resonance condition, to apparently experimentally accessible frequencies for which no new photons are generated. We show how this low-frequency resonance leads to the generation of entanglement between existing and previously non-entangled cavity modes. The oscillating cavity can be shown to behave like a generalised beam-splitter, thus performing an essential quantum gate functionality. We then discuss the possibility of performing actual experiments with mechanically oscillating optical cavities.

(1+1).— We first consider the simplified case of a cavity in $(1 + 1)$-dimensional Minkowski spacetime. The cavity is assumed mechanically rigid, maintaintaining constant length L in its instantaneous rest frame. The proper acceleration at the centre of the cavity is denoted by $a(\tau)$, where τ is the proper time. To maintain rigidity, the acceleration must be bounded by $|a(\tau)|L/c^2 < 2$ [11]. From now on we set $c = \hbar = 1$.

The cavity contains a real scalar field ϕ of mass $\mu_0 > 0$, with Dirichlet boundary conditions. When the cavity is inertial, the field has a standard orthonormal basis of positive frequency mode functions, with the nondenegenerate angular frequencies $\omega_n = \sqrt{M^2 + \pi^2 n^2}/L$, $n = 1, 2, \ldots$, where $M = \mu_0 L$. We denote by α and β the Bogoliubov coefficient matrices between the standard bases in an inertial initial region and a final inertial region, in the conventions of [11–13]. We write

$U = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix}$, so that the composition of Bogoliubov transformations is matrix multiplication of the corresponding U-matrices, and the Bogoliubov identities [13] read $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = U \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} U^\dagger$.

When the acceleration between the initial and final inertial regions is uniform and lasts for proper time $\bar\tau$, we have [11] $U_h(\bar\tau) = K_h^{-1}\tilde Z_h(\bar\tau)K_h$, where $K_h = \begin{pmatrix} {}_{\circ}\alpha_h & {}_{\circ}\beta_h \\ {}_{\circ}\beta_h & {}_{\circ}\alpha_h \end{pmatrix}$, $\tilde Z_h(\bar\tau) = \begin{pmatrix} Z_h(\bar\tau) & 0 \\ 0 & Z_h^*(\bar\tau) \end{pmatrix}$, $Z_h(\bar\tau) = \mathrm{diag}\big(e^{i\Omega_1(h)\bar\tau}, e^{i\Omega_2(h)\bar\tau}, \cdots\big)$, $\Omega_n(h)$ are the angular frequencies during the acceleration, ${}_{\circ}\alpha_h$ and ${}_{\circ}\beta_h$ are the Bogoliubov coefficient matrices from the initial inertial segment to the uniformly accelerated segment, and the acceleration has been encoded in the dimensionless parameter $h = aL$.

For accelerations that may vary arbitrarily between the initial time τ_0 and final time τ, $U(\tau,\tau_0)$ is given by the limit of $U_{h_N}(\bar\tau_N)U_{h_{N-1}}(\bar\tau_{N-1})\cdots U_{h_2}(\bar\tau_2)U_{h_1}(\bar\tau_1)$ as $N\to\infty$, such that $\tau-\tau_0 = \sum_{k=1}^N \bar\tau_k$ is fixed and each $\bar\tau_k \to 0$. As an infinitesimal increase in τ amounts to multiplying $U(\tau,\tau_0)$ from the left by $U_h(\bar\tau)$ with infinitesimal $\bar\tau$, $U(\tau,\tau_0)$ satisfies the differential equation

$$\dot U(\tau,\tau_0) = iK_{h(\tau)}^{-1}\tilde\Omega_{h(\tau)}K_{h(\tau)}U(\tau)\,, \tag{1}$$

where $\tilde\Omega_{h(\tau)} = \begin{pmatrix} \Omega_{h(\tau)} & 0 \\ 0 & -\Omega_{h(\tau)} \end{pmatrix}$, $\Omega_{h(\tau)} = \mathrm{diag}\big(\Omega_1\big(h(\tau)\big), \Omega_2\big(h(\tau)\big), \cdots\big)$, and the overdot denotes derivative with respect to τ. The solution is

$$U(\tau_f,\tau_0) = T\exp\left(i\int_{\tau_0}^{\tau_f} K_{h(\tau)}^{-1}\tilde\Omega_{h(\tau)}K_{h(\tau)}\,d\tau\right)\,, \tag{2}$$

where T denotes the time-ordered exponential.

To summarise: the Bogoliubov transformation between the inertial initial segment ending at proper time τ_0 and the final inertial segment starting at proper time τ_f is given by (2). $h(\tau)$ may vary arbitrarily for $\tau_0 \le \tau \le \tau_f$: in particular, no small acceleration approximation has been made. For piecewise constant $h(\tau)$, (2) reduces to a product of the matrices $U_h(\bar\tau_k)$ from each constant h segment.

A direct consequence of (2) is that the Bogoliubov coefficients evolve by pure phases over any time interval in which h is constant. Particles in the cavity are hence created by *changes* in the acceleration, not by acceleration itself, as can be argued on general adiabaticity grounds [14, 15]. The cavity is in this respect similar to a single accelerating mirror, which excites photons from the vacuum only when its acceleration is non-uniform [6].

(1+1): small accelerations.— At small accelerations, $\Omega_n(h)$, ${}_{\circ}\alpha_h$ and ${}_{\circ}\beta_h$ have the expansions

$$\Omega_n/\omega_n = 1 + O(h^2)\ , \quad n = 1,2,\ldots, \tag{3a}$$

$${}_{\circ}\alpha_h = 1 + h\hat\alpha + O(h^2), \quad {}_{\circ}\beta_h = h\hat\beta + O(h^2), \tag{3b}$$

where the elementary, M-dependent expressions for $\hat\alpha_{mn}$ and $\hat\beta_{mn}$ are given in [11]. $\hat\alpha$ is antisymmetric and $\hat\beta$ is symmetric, and both vanish when $m-n$ is even.

We seek $U(\tau,\tau_0)$ in the form

$$\alpha = e^{i\boldsymbol{\omega}(\tau-\tau_0)}\big(1 + \hat A + O(h^2)\big), \tag{4a}$$

$$\beta = e^{i\boldsymbol{\omega}(\tau-\tau_0)}\hat B + O(h^2), \tag{4b}$$

where $\boldsymbol{\omega} = \mathrm{diag}(\omega_1,\omega_2,\cdots)$ and $\hat A$ and $\hat B$ are of first order in h. Using (1) and (3), we find

$$\dot{\hat A}_{mn} = ih(\tau)\,(\omega_m-\omega_n)\,e^{-i(\omega_m-\omega_n)(\tau-\tau_0)}\,\hat\alpha_{mn}\ , \tag{5a}$$

$$\dot{\hat B}_{mn} = ih(\tau)\,(\omega_m+\omega_n)\,e^{-i(\omega_m+\omega_n)(\tau-\tau_0)}\,\hat\beta_{mn}\ . \tag{5b}$$

Once $h(\tau)$ is specified, $\hat A_{mn}$ and $\hat B_{mn}$ may be found from (5) by quadratures.

Three observations are immediate from (5).

First, $\hat A_{mn}$ and $\hat B_{mn}$ with even $m-n$ remain vanishing, regardless of the acceleration.

Second, in the special case of piecewise constant h, the changes in the magnitudes of $\hat A_{mn}$ and $\hat B_{mn}$ come entirely from the discontinuous jumps in h, as observed in [11]. However, if the jumps are replaced by a continuous evolution that is slow compared with the oscillating factors in (5), the changes in the magnitudes of $\hat A_{mn}$ and $\hat B_{mn}$ are significantly smaller. The assumption of rapidly-changing acceleration is hence essential for the non-periodic travel scenario results obtained in [11, 16, 17]. We emphasise that no such rapid changes are involved in the experimental scenario that will be considered below.

Third, if h is sinusoidal and its angular frequency ω_r equals the angular frequency of an oscillating factor in (5), the corresponding Bogoliubov coefficient will grow linearly in time. These resonance conditions read

$$\text{for } \hat A_{mn} \text{ with odd } m-n\text{:}\quad \omega_r = |\omega_m-\omega_n|\,, \tag{6a}$$

$$\text{for } \hat B_{mn} \text{ with odd } m-n\text{:}\quad \omega_r = \omega_m+\omega_n\,. \tag{6b}$$

The particle creation resonance (6b) is well known in the DCE literature [3, 4, 7, 10, 18–26]. The mode mixing resonance (6a) has been noted [10, 18–23] but seems to have received less attention due to the fact that the main effect of interest has been particle creation.

Mode mixing without particle creation is known in quantum optics as a passive transformation [27], implemented experimentally by passive optical elements such as beam splitters and phase plates. The oscillating cavity can hence theoretically be tuned to act as a beam splitter — a known quantum gate in continuous variable systems. Moreover, the entangling power of passive transformations is well understood [17, 28–30] and is directly applicable to our system. For example, the mode mixing generates entanglement from an initial Gaussian state only if this state is squeezed [29, 30]. We emphasise that while two-mode squeezing gates [12, 29, 30] and

other multipartite gates [31] can be implemented by the particle creation resonance (6b), mode-mixing gates are generated by the resonance (6a) even when no particle creation is present.

(3+1).— Let now ϕ be a real scalar field of mass $\mu \geq 0$ in a cavity in $(3+1)$-dimensional Minkowski space, with Dirichlet conditions. The inertial cavity is a rectangular parallelepiped with fixed edge lengths L_x, L_y and L_z, and a standard basis of orthonormal field modes is indexed by triples (m, n, p) of positive integers, such that the angular frequencies are $\omega_{mnp} = \sqrt{\mu^2 + (\pi m/L_x)^2 + (\pi n/L_y)^2 + (\pi p/L_z)^2}$.

Acceleration in the cavity's three principal directions can be treated as $(1+1)$-dimensional, with the inert transverse quantum numbers just contributing to the effective mass. Acceleration of unrestricted magnitude and direction would require new input regarding how the shape of the cavity responds to such acceleration. To *linear* order in the acceleration, however, boosts commute, and we can treat acceleration as a vector superposition of accelerations in the three principal directions in the cavity's instantaneous rest frame.

To find the $(3+1)$ Bogoliubov matrices, we first write the $(1+1)$ matrices $\hat{\alpha}$ and $\hat{\beta}$ in (3b) as $\hat{\alpha}(M)$ and $\hat{\beta}(M)$, indicating explicitly the dependence on the dimensionless mass parameter M [11]. We then define

$$ {}_x\hat{\alpha}_{mnp,m'n'p'} = \delta_{nn'}\delta_{pp'}\,\hat{\alpha}_{mm'}\left(\sqrt{\mu^2 L_x^2 + \pi^2 n^2 (L_x/L_y)^2 + \pi^2 p^2 (L_x/L_z)^2}\right), \tag{7a} $$

$$ {}_y\hat{\alpha}_{mnp,m'n'p'} = \delta_{mm'}\delta_{pp'}\,\hat{\alpha}_{nn'}\left(\sqrt{\mu^2 L_y^2 + \pi^2 m^2 (L_y/L_x)^2 + \pi^2 p^2 (L_y/L_z)^2}\right), \tag{7b} $$

$$ {}_z\hat{\alpha}_{mnp,m'n'p'} = \delta_{mm'}\delta_{nn'}\,\hat{\alpha}_{pp'}\left(\sqrt{\mu^2 L_z^2 + \pi^2 m^2 (L_z/L_x)^2 + \pi^2 n^2 (L_z/L_y)^2}\right), \tag{7c} $$

and we define analogously the matrices ${}_x\hat{\beta}$, ${}_y\hat{\beta}$ and ${}_z\hat{\beta}$. Writing $\boldsymbol{\omega} = \mathrm{diag}(\omega_{mnp})$ and defining $\hat{A}$ and $\hat{B}$ as in (4), equations (5) generalise to

$$ \dot{\hat{A}} = i \sum_j a_j(\tau) L_j \, e^{-i\boldsymbol{\omega}(\tau-\tau_0)} \, [\boldsymbol{\omega}, {}_j\hat{\alpha}] \, e^{i\boldsymbol{\omega}(\tau-\tau_0)} \,, \tag{8a} $$

$$ \dot{\hat{B}} = i \sum_j a_j(\tau) L_j \, e^{-i\boldsymbol{\omega}(\tau-\tau_0)} \, \{\boldsymbol{\omega}, {}_j\hat{\beta}\} \, e^{-i\boldsymbol{\omega}(\tau-\tau_0)} \,, \tag{8b} $$

where $[\cdot,\cdot]$ is the commutator, $\{\cdot,\cdot\}$ is the anticommutator and $\big(a_x(\tau), a_y(\tau), a_z(\tau)\big)$ is the acceleration three-vector in the cavity's instantaneous rest frame. Once the acceleration is specified, $\hat{A}$ and $\hat{B}$ may be found from (8) by quadratures.

For sinusoidal acceleration with angular frequency ω_r, the resonance condition is

$$ \text{for } \hat{A}_{mnp,m'n'p'}\text{:} \quad \omega_r = |\omega_{mnp} - \omega_{m'n'p'}| \,, \tag{9a} $$

$$ \text{for } \hat{B}_{mnp,m'n'p'}\text{:} \quad \omega_r = \omega_{mnp} + \omega_{m'n'p'} \,, \tag{9b} $$

where in each case the difference in the quantum number in the direction of oscillation needs to be odd.

Desktop experiment.— The mode mixing resonance angular frequency (9a) can be significantly lower than the frequencies of the individual cavity modes. We outline an experimental scenario that optimises this lowering.

Setting $\mu = 0$, we assume that the quanta in the cavity have wavelength $\lambda \ll \min(L_x, L_y, L_z)$ and have their momenta aligned close to the z-direction, so that $(2/\lambda)^2 \approx (p/L_z)^2 \gg (m/L_x)^2 + (n/L_y)^2$ and $\omega_{mnp} \approx 2\pi/\lambda + \frac{1}{4}\pi\lambda\big[(m/L_x)^2 + (n/L_y)^2\big]$. We let the cavity undergo linear or circular harmonic oscillation orthogonal to the z-direction, with amplitude r_x (r_y) in the x-direction (y-direction). For motion in x-direction, the mode mixing resonance angular frequency (9a) between modes m and m', with $m - m'$ odd, is

$$ \omega_r \approx \tfrac{1}{4}\pi\lambda L_x^{-2} \left| m^2 - (m')^2 \right| , \tag{10} $$

and it follows using (5a), (8a) and the formulas for $\hat{\alpha}(M)$ [11] that the mode mixing growth rate is

$$ \tfrac{d}{d\tau} |\hat{A}_{\mathrm{res}}| \approx \tfrac{1}{2}\pi m m' r_x \lambda L_x^{-3} \,. \tag{11} $$

The lowest resonance occurs for $m = 1$ and $m' = 2$. Similar formulas ensue for the y-resonance, and for circular motion both resonances are present.

As an experimental scenario, we first trap one or more quanta in the cavity, in modes whose momenta are aligned close to the z-direction. After a period of linear or circular oscillation perpendicular to the z-direction, a measurement on the quantum state of the cavity is performed. We assume that the resonance mode mixing dominates any effects due to the initial trapping and the final releasing of the quanta.

We choose for example $\lambda = 600\,\mathrm{nm}$ and $L_x = L_y = 1\,\mathrm{cm}$. The lowest resonance angular frequency is $\omega_r \approx 1.4 \times 10^{-2}\,\mathrm{m}^{-1} \approx 4.2 \times 10^6\,\mathrm{s}^{-1}$, corresponding to an oscillation frequency $0.7\,\mathrm{MHz}$. With amplitude $1\,\mu\mathrm{m}$, (11) and its y-counterpart gives $\frac{d}{d\tau}|\hat{A}_{\mathrm{res}}| \approx 6 \times 10^2\,\mathrm{s}^{-1}$, so that

the mode mixing coefficient grows to order unity within a millisecond. For longer periods the linear perturbation theory is no longer quantitatively reliable, but there is no reason to expect the mode mixing coefficients to decrease.

For linear motion, oscillation of micron amplitude at megahertz frequency may be achievable by using ultrasound to accelerate the cavity. Storing the quantum in the cavity for a millisecond could be challenging although recent achievements indicate that it may be feasible [32].

For circular motion, the threshold angular velocity $\omega_r \approx 4.2 \times 10^6\,\mathrm{s}^{-1} \approx 4 \times 10^7\,\mathrm{rpm}$ exceeds the angular velocity $1.5 \times 10^5\,\mathrm{rpm}$ achieved by medical ultracentrifuges [33], although by an amount that may possibly be bridged by a specifically designed system.

Conclusions.— We have quantised a scalar field in a rectangular cavity that is accelerated arbitrarily in $(3+1)$-dimensional Minkowski spacetime, in the limit of small accelerations but arbitrary velocities and travel times. The Bogoliubov coefficients were expressed as explicit quadratures. For linear or circular periodic motions, we identified a configuration in which the mode mixing resonance frequency is significantly below the frequencies of the cavity modes. If quantisation of the electromagnetic field in an accelerated cavity is qualitatively similar, the mode mixing effects are within the reach of a desktop experiment that appears achievable with current technology in its mechanical aspects, if not yet in the storage capabilities required of a mechanically oscillating optical cavity.

We also anticipate that the particle creation and mode mixing effects are not qualitatively sensitive to the detailed shape of the cavity, and this freedom could be utilised in the development of a concrete laboratory implementation. The experimental prospects could be further improved by filling the cavity with a medium that slows light down [34].

We underline that our experimental scenario does not involve significant particle creation. Nevertheless, it involves significant mode mixing. This mixing acts as a beam splitter quantum gate, creating or degrading entanglement in situations where particles are initially present. We anticipate that observations of entanglement will generally provide opportunities for experimental verification of the DCE that are complementary to observations of fluxes or particle numbers [8].

Acknowledgments.— We thank Nico Giulini and Bill Unruh for invaluable discussions. We also thank Iacopo Carusotto, Fay Dowker, Gary Gibbons, Chris Fewster, Vladimir Man'ko, Carlos Sabín, Ralf Schützhold and Matt Visser for helpful comments. J. L. thanks Gabor Kunstatter for hospitality at the University of Winnipeg and the organisers of the "Bits, Branes, Black Holes" programme for hospitality at the Kavli Institute for Theoretical Physics, University of California at Santa Barbara, supported in part by the National Science Foundation under Grant No. NSF PHY11-25915. D.F. acknowledges financial support from EPSRC project EP/J00443X/1. I.F. acknowledges financial support from EPSRC [CAF Grant EP/G00496X/2]. J.L. acknowledges financial support from STFC [Theory Consolidated Grant ST/J000388/1].

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